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An isogeometric approach for size-dependent buckling analysis of FGM nanoplates

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Abstract. In this paper, a suitable and simple computational formulation based on Isogeometric Analysis (IGA) integrated with higher-order shear deformation theory (HSDT) is introduced for size-dependent buckling analysis of functionally graded material (FGM) nanoplates. The material properties of FGM based on the Mori–Tanaka schemes and the rule of mixture are used. The differential nonlocal equations are utilized to take into account size effects. The nonlocal governing equations are approximated according to IGA based on HSDT, which satisfies naturally the higher-order derivatives continuity requirement in weak form of FGM nanoplates. The effect of nonlocal approach on the behaviors of the FGM nanoplates with several volume fraction exponents is investigated to show the reliability of the proposed method.

1. Introduction

There has recently been a fast growth in applications of nanoscale structures, which are primarily concerned with fabrication of FGMs. FGM nano-plates, as specific nanostructures, have been applied in the engineering and technology sectors. Therefore, the understanding of the behaviours of the FGM nano-plates is essential for the development of nanostructures because of their huge potential applications in the real life. In continuum mechanics, one of the well-known theories, which includes small scale effects with a good agreement and accuracy with molecular dynamics (MD) simulations, is the nonlocal continuum theory of Eringen [1]. Finite Element method (FEM) is a very well know numerical techniques and has been used for a wide range of applications [2-15]. Using FEM, Demir and Civalek [16] investigated small scale effects on vibration response of microtubules. The free vibration and buckling analyses of FG nano-plate using Navier solution subjected to thermal load were reported in Ref. [17]. Effect the beam thickness reduction on bending behaviour of micro-beams was observed in Ref. [18]. The mechanical behaviour for homogenous nonlinear micro beams [19], FG nonlinear micro beams [20]. A nonlinear micro beams model based on the strain gradient elasticity is introduced in Refs. [21, 22]. Size-dependent free analysis of FGM square plate based on FSDT was reported by Natarajan et al. [23]. Size-dependent analysis of FG nano-plates using IGA based on quasi-3D theory is recently examined in Ref [24]. Recently, Phung-Van et al. studied static and free vibration analyses of FG-CNTRC nano-plates [25] and nonlinear transient analysis of FGM nano-plates [26]. Furthermore, mechanical behaviours of FGM composite plates based on the local continuum theory have recently been published in Refs. [27-43]. This paper hence aims to develop a size-dependent buckling analysis of FGM nano-plates by a combination of IGA and the nonlocal continuum theory based on HSDT. In particular, we show that IGA based on HSDT fulfilling C2-continuity requirements can easily achieve the higher-order derivatives in the framework of the



nonlocal continuum theory, which is of interest in this study. Size effects based on Eringen [1] in the differential nonlocal equations are performed. The effect of nonlocal approach on the behaviors of the FGM nanoplates with various volume fraction exponents are discussed in details.

2. Functional graded materials

A functionally graded material nanoplate made of ceramic and metal is considered in this research. The properties materials based on the rule of mixture can be given as:

$$P(z) = (P_c - P_m)V_c + P_m \quad ; \quad V_c = \left(\frac{1}{2} + \frac{z}{t}\right)^n \quad (n \geq 0) \quad ; \quad V_m = 1 - V_c \quad (1)$$

where m and c represent the metal and ceramic constituents, respectively, P refers to the effective material properties including the thermal conductivity k , ν Poisson's ratio, ρ density, E Young's modulus and α thermal expansion. V_m and V_c are the volume fraction of the metal and ceramic, respectively, z is the thickness coordinate of plate and varies from $-t/2$ to $t/2$ and n is the volume fraction exponent.

3. Theoretical formulation

3.1 Nonlocal elasticity theory

Eringen [1] exhibited an equivalent form of differential equations of nonlocal stress at any points \mathbf{x} as follows:

$$(1 - \mu \nabla^2) \sigma_{ij} = t_{ij} \quad (2)$$

where $\mu = e_0 l$, $0 \leq \mu \leq 4$ is the small-scale effect; l is an internal characteristic length; e_0 is material constant and $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the Laplace operator.

3.2 Displacement field

The displacement field can be expressed as follows [44]:

$$\begin{aligned} u(x, y, z) &= u_0 + z\beta_x + cz^3(\beta_x + w_{,x}) \\ v(x, y, z) &= v_0 + z\beta_y + cz^3(\beta_y + w_{,y}), \quad (-h/2 \leq z \leq h/2) \\ w(x, y, z) &= w_0 \end{aligned} \quad (3)$$

where $c = 4/3h^2$.

The Green strain-displacement relations are now given as

$$\boldsymbol{\varepsilon} = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy}]^T = \boldsymbol{\varepsilon}_m + z\boldsymbol{\kappa}_1 + z^3\boldsymbol{\kappa}_2 \quad ; \quad \boldsymbol{\gamma} = [\gamma_{xz} \quad \gamma_{yz}]^T = \boldsymbol{\varepsilon}_s + z^2\boldsymbol{\kappa}_s \quad (4)$$

where

$$\boldsymbol{\varepsilon}_m = \begin{bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{bmatrix}; \quad \boldsymbol{\kappa}_1 = \begin{bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{bmatrix}; \quad \boldsymbol{\kappa}_2 = c \begin{bmatrix} \beta_{x,x} + w_{0,xx} \\ \beta_{y,y} + w_{0,yy} \\ \beta_{x,y} + \beta_{y,x} + 2w_{0,xy} \end{bmatrix}; \quad \boldsymbol{\varepsilon}_s = \begin{bmatrix} \beta_x + w_{0,x} \\ \beta_y + w_{0,y} \end{bmatrix}; \quad \boldsymbol{\kappa}_s = 3c \begin{bmatrix} \beta_x + w_{0,x} \\ \beta_y + w_{0,y} \end{bmatrix} \quad (5)$$

3.3 Isogeometric analysis

The displacement field of the plate can be expressed as:

$$\mathbf{u}^h(\xi, \eta) = \sum_{I=1}^{m \times n} R_I(\xi, \eta) \mathbf{d}_I \quad (6)$$

where $\mathbf{d}_I = [u_{0I} \quad v_{0I} \quad \beta_{xI} \quad \beta_{yI} \quad w_I]^T$ is the vector of degrees of freedom associated with the control point I , and R_I is the shape function as defined in Ref [45].

The in-plane and shear strains can be expressed as:

$$[\boldsymbol{\varepsilon}_0 \quad \boldsymbol{\kappa}_1 \quad \boldsymbol{\kappa}_2 \quad \boldsymbol{\kappa}_s]^T = \sum_{I=1}^{m \times n} [\mathbf{B}_I^m \quad \mathbf{B}_I^{b1} \quad \mathbf{B}_I^{b2} \quad \mathbf{B}_I^s]^T \mathbf{d}_I = \sum_{I=1}^{m \times n} [\mathbf{B}_I^{mb} \quad \mathbf{B}_I^s]^T \mathbf{d}_I \quad (7)$$

where

$$\mathbf{B}_I^m = \begin{bmatrix} R_{I,x} & 0 & 0 & 0 & 0 \\ 0 & R_{I,y} & 0 & 0 & 0 \\ R_{I,y} & R_{I,x} & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_I^{b1} = - \begin{bmatrix} 0 & 0 & R_{I,xx} & 0 & 0 \\ 0 & 0 & R_{I,yy} & 0 & 0 \\ 0 & 0 & 2R_{I,xy} & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_I^{b2} = \begin{bmatrix} 0 & 0 & 0 & R_{I,x} & 0 \\ 0 & 0 & 0 & 0 & R_{I,y} \\ 0 & 0 & 0 & R_{I,y} & R_{I,x} \end{bmatrix}, \quad \mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & 0 & R_I & 0 \\ 0 & 0 & 0 & 0 & R_I \end{bmatrix}$$
(8)

The governing algebraic equations for buckling analysis can be obtained as follows

$$(\mathbf{K} - \lambda \mathbf{K}_g) \mathbf{d} = \mathbf{0}$$
(9)

where

$$\mathbf{K} = \int_{\Omega} \left[(\mathbf{B}^{mb})^T \mathbf{D}_{mb} \mathbf{B}_I^{mb} + (\mathbf{B}^s)^T \mathbf{D}_s \mathbf{B}^s \right] d\Omega$$

$$\mathbf{K}_g = \int_{\Omega} \left[(\mathbf{B}^g)^T - \lambda \nabla^2 (\mathbf{B}^g)^T \right] \mathbf{N}_0 \mathbf{B}^g d\Omega$$
(10)

in which

$$\mathbf{D}_{mb} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{E} \\ \mathbf{B} & \mathbf{D} & \mathbf{F} \\ \mathbf{E} & \mathbf{F} & \mathbf{H} \end{bmatrix}; \quad \mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & R_{I,x} & 0 & 0 \\ 0 & 0 & R_{I,y} & 0 & 0 \end{bmatrix}$$
(11)

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{-h/2}^{h/2} (1, z, z^2, f(z), zf(z), f^2(z)) C_{ij} dz, \quad i, j = 1, 2, 6$$

$$D_{sij} = \int_{-h/2}^{h/2} [f'(z)]^2 C_{ij} dz, \quad i, j = 4, 5$$

It can be seen that second and third order derivatives are included in stiffness and buckling matrixes, respectively. The order of the basic functions is at least three. Hence, it is clear that NURBS basic function is the most suitable for calculating size-dependent analysis of nanoplate structures.

4. Numerical results

In this study, the critical buckling loads $\bar{P}_{cr} = P_{cr} R^2 / D_m$ of Al/Al₂O₃ circular plate (radius $R = 10$, $h = 0.34$) are performed. Table 1 shows Non-dimensional buckling load of simply supported and clamped circle nanoplate. It can be observed that the present results match well with the reference solution [24].

Table 1. Material properties of FGM plates.

BCs	μ	Model	n				
			0	1	2	5	10
SSSS	0	Quasi-3D [24]	23.1059	10.0048	8.5094	7.0907	6.1583
		HSDT-Present	22.7565	9.6263	8.1811	6.8955	6.0381
	1	Quasi-3D [24]	22.1473	9.5755	8.1448	6.7891	5.8991
		HSDT-Present	21.8371	9.2172	7.8328	6.6045	5.7871
CCCC	0	Quasi-3D [24]	82.3402	33.5902	28.5574	24.4165	21.6023
		HSDT-Present	79.3204	31.5156	26.7567	23.1807	20.7108
	1	Quasi-3D [24]	71.7287	29.2705	24.8837	21.2721	18.8188
		HSDT-Present	69.1652	27.4808	23.3311	20.2129	18.0593

5. Conclusions

In this paper, a simple and effective approach using IGA based on HSDT for size-dependent buckling analysis of FGM nanoplates. The material properties of FGM based on the Mori–Tanaka schemes and the rule of mixture are considered. The differential nonlocal equations are utilized to take into account effect of the size-dependent. The buckling nonlocal governing equations of motion is approximated according to IGA based on HSDT, which satisfies naturally the 3rd derivatives of displacement field. The effects of volume fraction exponent and small scale parameter on nonlinear transient of FGM nanoplates are also performed. Numerical results proved high accuracy and reliability of the present method in comparison with other available numerical approaches.

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